Paper Reference(s)

6663/01

Edexcel GCE

Core Mathematics C1

Advanced Subsidiary

Wednesday 16 May 2012 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Calculators may NOT be used in this examination.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C1), the paper reference (6663), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 10 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P40684A

1. Find

$$\int \left(6x^2 + \frac{2}{x^2} + 5 \right) \mathrm{d}x,$$

giving each term in its simplest form.

(4)

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2. (*a*) Evaluate $(32)^{\frac{3}{5}}$, giving your answer as an integer.

(2)

(*b*) Simplify fully $\left(\frac{25x^4}{4} \right)^{-\frac{1}{2}}$.

(2)

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3. Show that $\frac{2}{\sqrt{12} - \sqrt{8}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers.

(5)

4.

$$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3.$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$, giving each term in its simplest form.

(4)

(*b*) Find $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$.

(2)

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5. A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3,$$

$$a_{n+1} = 2a_n - c, \qquad (n \geq 1),$$

where c is a constant.

(*a*) Write down an expression, in terms of c, for a_2.

(1)

(*b*) Show that $a_3 = 12 - 3c$.

(2)

Given that $\sum_{i=1}^{4} a_i \geq 23$,

(*c*) find the range of values of c.

(4)

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6. A boy saves some money over a period of 60 weeks. He saves 10p in week 1, 15p in week 2, 20p in week 3 and so on until week 60. His weekly savings form an arithmetic sequence.

(*a*) Find how much he saves in week 15.

(2)

(*b*) Calculate the total amount he saves over the 60 week period.

(3)

The boy's sister also saves some money each week over a period of m weeks. She saves 10p in week 1, 20p in week 2, 30p in week 3 and so on so that her weekly savings form an arithmetic sequence. She saves a total of £63 in the m weeks.

(*c*) Show that

$$m(m + 1) = 35 \times 36.$$

(4)

(*d*) Hence write down the value of m.

(1)

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7. The point P (4, –1) lies on the curve C with equation $y = \mathrm{f}(x)$, $x > 0$, and $\mathrm{f}(x)$, $x > 0$, and

$$\mathrm{f}'(x) = \frac{1}{2}x - \frac{6}{\sqrt{x}} + 3.$$

(*a*) Find the equation of the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are integers.

(4)

(*b*) Find $\mathrm{f}(x)$.

(4)

8.

$$4x - 5 - x^2 = q - (x + p)^2,$$

where p and q are integers.

(*a*) Find the value of p and the value of q.

(3)

(*b*) Calculate the discriminant of $4x - 5 - x^2$.

(2)

(*c*) Sketch the curve with equation $y = 4x - 5 - x^2$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(3)

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9. The line L_1 has equation $4y + 3 = 2x$.

The point A $(p, 4)$ lies on L_1.

(*a*) Find the value of the constant p.

(1)

The line L_2 passes through the point C $(2, 4)$ and is perpendicular to L_1.

(*b*) Find an equation for L_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

The line L_1 and the line L_2 intersect at the point D.

(*c*) Find the coordinates of the point D.

(3)

(*d*) Show that the length of CD is $\frac{3}{2}\sqrt{5}$.

(3)

A point B lies on L_1 and the length of $AB = \sqrt{80}$.

The point E lies on L_2 such that the length of the line CDE = 3 times the length of CD.

(*e*) Find the area of the quadrilateral $ACBE$.

(3)

10.

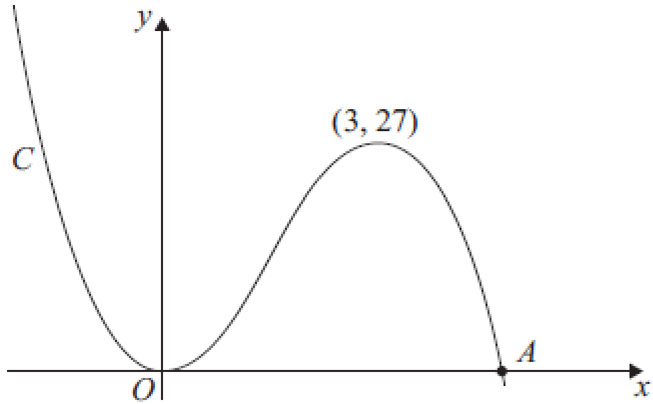


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$, where

$$f(x) = x^2(9 - 2x).$$

There is a minimum at the origin, a maximum at the point (3, 27) and C cuts the x-axis at the point A.

(*a*) Write down the coordinates of the point A.

(1)

(*b*) On separate diagrams sketch the curve with equation

(i) $y = f(x + 3)$,

(ii) $y = f(3x)$.

On each sketch you should indicate clearly the coordinates of the maximum point and any points where the curves cross or meet the coordinate axes.

(6)

The curve with equation $y = f(x) + k$, where k is a constant, has a maximum point at (3, 10).

(*c*) Write down the value of k.

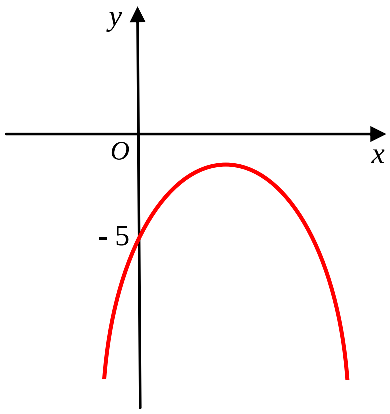
(1)

TOTAL FOR PAPER: 75 MARKS

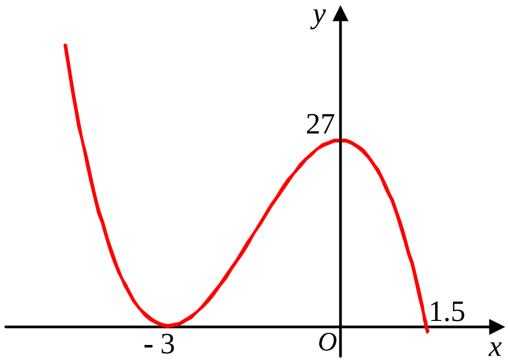
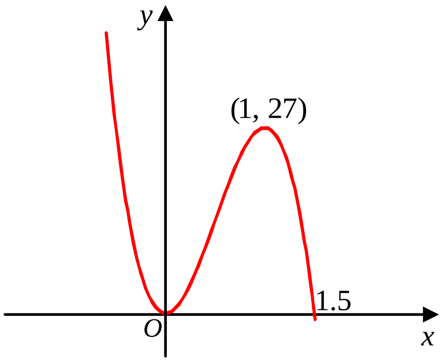
END

Question Number	Scheme	Marks
1.	$\left\{\int\left(6x^2+\frac{2}{x^2}+5\right)\mathrm{d}x\right\} = \frac{6x^3}{3}+\frac{2x^{-1}}{-1}+5x\,(+c)$	M1 A1
	$= 2x^3 - 2x^{-1}\,;+5x+c$	A1; A1
		4
2. (a)	$\left\{(32)^{\frac{3}{5}}\right\} = \left(\sqrt[5]{32}\right)^3$ or $\sqrt[5]{(32)^3}$ or 2^3 or $\sqrt[5]{32768}$	M1
	$= 8$	A1
		[2]
(b)	$\left\{\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}\right\} = \left(\frac{4}{25x^4}\right)^{\frac{1}{2}}$ or $\left(\frac{5x^2}{2}\right)^{-1}$ or $\frac{1}{\left(\frac{25x^4}{4}\right)^{\frac{1}{2}}}$	M1
	$= \frac{2}{5x^2}$ or $\frac{2}{5}x^{-2}$	A1
		[2] **4**
3.	$\left\{\frac{2}{\sqrt{12}-\sqrt{8}}\right\} = \frac{2}{\left(\sqrt{12}-\sqrt{8}\right)}\times\frac{\left(\sqrt{12}+\sqrt{8}\right)}{\left(\sqrt{12}+\sqrt{8}\right)}$ Writing this is sufficient for M1.	M1
	$= \frac{\left\{2\left(\sqrt{12}+\sqrt{8}\right)\right\}}{12-8}$ For $12-8$. This mark can be implied.	A1
	$= \frac{2\left(2\sqrt{3}+2\sqrt{2}\right)}{12-8}$	B1 B1
	$= \sqrt{3}+\sqrt{2}$	A1 **cso**
		5
	$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$	
4. (a)	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x}=\right\}\ 5(3)x^2 - 6\left(\frac{4}{3}\right)x^{\frac{1}{3}} + 2$	M1
	$= 15x^2 - 8x^{\frac{1}{3}} + 2$	A1 A1 A1
		[4]
(b)	$\left\{\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\right\}\ 30x - \frac{8}{3}x^{-\frac{2}{3}}$	M1 A1
		[2] **6**

Question Number	Scheme	Marks
	$a_1 = 3,\ a_{n+1} = 2a_n - c\ ,\quad n \geq 1\ ,\ c$ is a constant	
5. (a)	$\{a_2 =\}\ \ 2\times 3 - c$ or $2(3) - c$ or $6 - c$	B1
		[1]
(b)	$\{a_3 =\}\ \ 2\times\left(\text{"}6 - c\text{"}\right) - c$	M1
	$= 12 - 3c$ **(*)**	A1 cso
		[2]
(c)	$a_4 = 2\times\left(\text{"}12 - 3c\text{"}\right) - c \qquad \{= 24 - 7c\}$	M1
	$\left\{\sum_{i=1}^{4} a_i =\right\}\ \ 3 + (6 - c) + (12 - 3c) + (24 - 7c)$	M1
	$\text{"}45 - 11c\text{"} \geq 23$ or $\text{"}45 - 11c\text{"} = 23$	M1
	$c \leq 2$ or $2 \geq c$	A1 cso
		[4]
		7
	Boy's Sequence: $10, 15, 20, 25, \ldots$	
6. (a)	$\{a = 10,\ d = 5 \Rightarrow T_{15} =\}\ a + 14d = 10 + 14(5); = 80$ or $0.1 + 14(0.05); = £0.80$	M1; A1
		[2]
(b)	$\{S_{60} =\}\ \dfrac{60}{2}\left[2(10) + 59(5)\right]$	M1 A1
	$= 30(315) = 9450$ or $£94.50$	A1
		[3]
	Boy's Sister's Sequence: $10, 20, 30, 40, \ldots$	
(c)	$\{a = 10,\ d = 10 \Rightarrow S_m =\}\ \dfrac{m}{2}\left(2(10) + (m-1)(10)\right) \qquad \left(\text{or } \dfrac{m}{2}\times 10(m+1) \text{ or } 5m(m+1)\right)$	M1 A1
	63 or $6300 = \dfrac{m}{2}\left(2(10) + (m-1)(10)\right)$	dM1
	$6300 = \dfrac{m}{2}(10)(m+1)$ or $12600 = 10m(m+1)$	
	$1260 = m(m+1)$	
	$35\times 36 = m(m+1)$ **(*)**	A1 **cso**
		[4]
(d)	$\{m =\}\ 35$	B1
		[1]
		10

Question Number	Scheme	Marks
	$P(4,-1)$ lies on C where $f'(x) = \frac{1}{2}x - \frac{6}{\sqrt{x}} + 3,\ x > 0$	
7. (a)	$f'(4) = \frac{1}{2}(4) - \frac{6}{\sqrt{4}} + 3\ ;\ = 2$	M1; A1
	T: $y - -1 = 2(x-4)$	dM1
	T: $y = 2x - 9$	A1
		[4]
(b)	$f(x) = \frac{x^{1+1}}{2(2)} - \frac{6x^{-\frac{1}{2}+1}}{(\frac{1}{2})} + 3x\ (+c)$ or equivalent.	M1 A1
	$\{f(4) = -1 \Rightarrow\}\ \frac{16}{4} - 12(2) + 3(4) + c = -1$	dM1
	$\{4 - 24 + 12 + c = -1 \quad \Rightarrow \quad c = 7\}$	
	So, $\{f(x) =\}\ \frac{x^2}{2(2)} - \frac{6x^{\frac{1}{2}}}{(\frac{1}{2})} + 3x + 7$	A1 cso
	$\left\{\text{NB: } f(x) = \frac{x^2}{4} - 12\sqrt{x} + 3x + 7\right\}$	**[4]**
		8
	$4x - 5 - x^2 = q - (x-p)^2,\ p,\ q$ are integers.	
8. (a)	$\{4x - 5 - x^2 =\}\ -\left[x^2 - 4x + 5\right] = -\left[(x-2)^2 - 4 + 5\right] = -\left[(x-2)^2 + 1\right]$	M1
	$= -1 - (x-2)^2$	A1 A1
		[3]
(b)	$\{"b^2 - 4ac" =\}\ 4^2 - 4(-1)(-5) \quad \{= 16 - 20\}$	M1
	$= -4$	A1
		[2]
(c)		
	Correct ∩ shape	M1
	Maximum **within** the 4th quadrant	A1
	Curve cuts through -5 or $(0, -5)$ marked on the y-axis	B1
		[3]
		8

Question Number	Scheme	Marks
	$L_1:\ 4y+3=2x \Rightarrow y=\frac{1}{2}x-\frac{3}{4};\quad A(p,4)$ lies on L_1.	
9. (a)	$\{p=\}\ 9\frac{1}{2}$ or $\frac{19}{2}$ or 9.5	B1
		[1]
(b)	$\{4y+3=2x\} \Rightarrow y=\frac{2x-3}{4} \Rightarrow m(L_1)=\frac{1}{2}$ or $\frac{2}{4}$	M1 A1
	So $m(L_2)=-2$	B1ft
	$L_2:\ y-4=-2(x-2)$	M1
	$L_2:\ 2x+y-8=0$ or $L_2:\ 2x+1y-8=0$	A1
		[5]
(c)	$\{L_1=L_2 \Rightarrow\}\ 4(8-2x)+3=2x$ or $-2x+8=\frac{1}{2}x-\frac{3}{4}$	M1
	$x=3.5,\ y=1$	A1, A1 cso
		[3]
(d)	$CD^2=("3.5"-2)^2+("1"-4)^2$	"M1"
	$CD=\sqrt{("3.5"-2)^2+("1"-4)^2}$	A1 ft
	$=\sqrt{1.5^2+3^2}=1.5\sqrt{1^2+2^2}=1.5\sqrt{5}$ or $\frac{3}{2}\sqrt{5}$ **(*)**	A1 **cso**
		[3]
(e)	Area = triangle ABC + triangle ABE	
	$=\frac{1}{2}\times\frac{3}{2}\sqrt{5}\times\sqrt{80}+\frac{1}{2}\times3\sqrt{5}\times\sqrt{80}$ Finding the area of any triangle.	M1
	$=\frac{3}{4}\sqrt{5}\times4\sqrt{5}+\frac{3}{2}\sqrt{5}\times4\sqrt{5}$	
	$=\frac{3}{4}(20)+\frac{3}{2}(20)$	B1
	$=45$	A1
		[3] **15**

Question Number	Scheme	Marks
10. (a)	{Coordinates of A are} $(4.5, 0)$ See notes below	B1 **[1]**
(b)(i)		
	Horizontal translation	M1
	-3 and their ft 1.5 on postitive x-axis	A1 ft
	Maximum at 27 marked on the y-axis	B1 **[3]**
(ii)		
	Correct shape, minimum at $(0, 0)$ and a maximum within the first quadrant.	M1
	1.5 on x-axis	A1 ft
	Maximum at $(1, 27)$	B1 **[3]**
(c)	$\{k =\}\ -17$	B1 **[1]**
		8